Classification

Statistical Natural Language Processing 1

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Seminar für Sprachwissenschaft

Winter Semester 2025/2026

version: 9ac64cd @2026-01-20

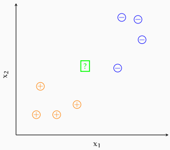
When/why do we do classification

- Is a given email spam or not?
- What is the gender of the author of a document?
- Is a product review positive or negative?
- Who is the author of a document?
- What is the subject of an article?
- ...

As opposed to regression, the outcome is a 'category'.

The task

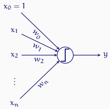
- Given a set of training data with (categorical) labels
- Train a model to predict future data points from the same distribution



Outline

- Perceptron
- Logistic regression
- Naive Bayes
- Multi-class strategies for binary classifiers
- Evaluation metrics for classification
- Brief notes on what we skipped

The perceptron



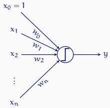
$$y = f\left(\sum_i^n w_i x_i\right)$$

where

$$f(x) = \begin{cases} +1 & \text{if } \sum_i^n w_i x_i > 0 \\ -1 & \text{otherwise} \end{cases}$$

Similar to the *intercept* in linear models, an additional input x_0 which is always set to one is often used (called *bias* in ANN literature)

The perceptron: in plain words



- Sum all input x_i weighted with corresponding weight w_i
- Classify the input using a threshold function

positive: the sum is larger than 0
negative: otherwise

Learning with perceptron

- We do not update the parameters if classification is correct
- For misclassified examples, we try to minimize

$$E(w) = -\sum_i \boldsymbol{w}\boldsymbol{x}_i y_i$$

where i ranges over all misclassified examples

- Perceptron algorithm updates the weights such that

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \eta \nabla E(\boldsymbol{w})$$
$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \eta \boldsymbol{x}_i y_i$$

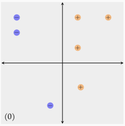
for misclassified examples. η is the learning rate

The perceptron algorithm

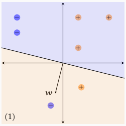
- The perceptron algorithm can be

online: update weights for a single misclassified example
batch: updates weights for all misclassified examples at once

- The perceptron algorithm converges to the global minimum if the classes are *linearly separable*
- If the classes are not linearly separable, the perceptron algorithm will not stop
- We do not know whether the classes are linearly separable or not before the algorithm converges
- In practice, one can set a stopping condition, such as
  - Maximum number iterations/updates
  - Number of misclassified examples
  - Number of iterations without improvement



1. Randomly initialize $\boldsymbol{w}$ (the decision boundary is orthogonal to $\boldsymbol{w}$)
2. Pick a misclassified example $\boldsymbol{x}_i$ add $y_i\boldsymbol{x}_i$ to $\boldsymbol{w}$
3. Set $\boldsymbol{w} \leftarrow \boldsymbol{w} + y_i\boldsymbol{x}_i$, go to step 2 until convergence



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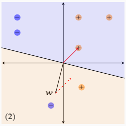

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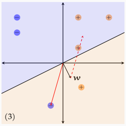

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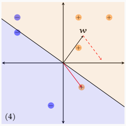

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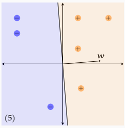

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Perceptron: a bit of history

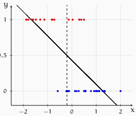
- The perceptron was developed in late 1950's and early 1960's (Rosenblatt, 1958)
- It caused excitement in many fields including computer science, artificial intelligence, cognitive science
- The excitement (and funding) died away in early 1970's (after the criticism by Minsky and Papert, 1969)
- The main issue was the fact that the perceptron algorithm cannot handle problems that are not linearly separable

Logistic regression

- Logistic *regression* is a *classification* method
- In logistic regression, we fit a model that predicts $P(y \mid x)$
- Logistic regression is an extension of linear regression
  - it is a member of the family of models called generalized linear models
- Typically formulated for binary classification, but it has a natural extension to multiple classes
- The multi-class logistic regression is often called *maximum-entropy model* (or max-ent) in the NLP literature

Data for logistic regression

an example with a single predictor



- Why not just use linear regression?
- What is $P(y \mid x = 2)$?
- Is RMS error appropriate?

Fixing the outcome: transforming the output variable

- The prediction we are interested in is $\hat{y} = P(y = 1|x)$
- We transform it with logit function:

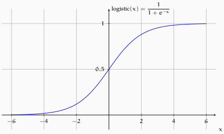
$$\text{logit}(\hat{y}) = \log \frac{\hat{y}}{1 - \hat{y}} = w_0 + w_1 x$$

- $\frac{\hat{y}}{1-\hat{y}}$ (odds) is bounded between 0 and ∞
- $\log \frac{\hat{y}}{1-\hat{y}}$ (log odds) is bounded between $-\infty$ and ∞
- we can estimate $\text{logit}(\hat{y})$ with regression, transform with the inverse of logit()

$$\hat{y} = \frac{e^{w_0 + w_1 x}}{1 + e^{w_0 + w_1 x}} = \frac{1}{1 + e^{-w_0 - w_1 x}}$$

which is called logistic (sigmoid) function

Logistic function



How to fit a logistic regression model

with maximum-likelihood estimation

$$P(y = 1 \mid \boldsymbol{x}) = p = \frac{1}{1 + e^{-\boldsymbol{w}\boldsymbol{x}}} \qquad P(y = 0 \mid \boldsymbol{x}) = 1 - p = \frac{e^{-\boldsymbol{w}\boldsymbol{x}}}{1 + e^{-\boldsymbol{w}\boldsymbol{x}}}$$

The likelihood of the training set is,

$$\mathcal{L}(\boldsymbol{w}) = \prod_i p^{y_i} (1 - p)^{1 - y_i}$$

In practice, we maximize log likelihood, or minimize '− log likelihood':

$$-\log \mathcal{L}(\boldsymbol{w}) = -\sum_i y_i \log p + (1 - y_i) \log(1 - p)$$

How to fit a logistic regression model (2)

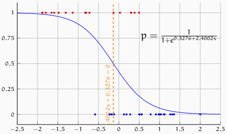
- Bad news: there is no analytic solution
- Good news: the (negative) log likelihood is a convex function
- We can use iterative methods such as *gradient descent* to find parameters that maximize the (log) likelihood
- Using gradient descent, we repeat

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \eta \nabla E(\boldsymbol{w})$$

until convergence, η is the *learning rate*

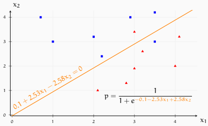
Example logistic-regression

back to the example with a single predictor



Another example

two predictors



Multi-class logistic regression

- Generalizing logistic regression to more than two classes is straightforward
- We estimate,

$$P(C_k \mid x) = \frac{e^{\boldsymbol{w}_k \boldsymbol{x}}}{\sum_j e^{\boldsymbol{w}_j \boldsymbol{x}}}$$

where C_k is the k^{th} class, j iterates over all classes.
- The function is called the *softmax* function, used frequently in neural network models as well
- This model is also known as *log-linear model*, *maximum entropy* model, or *Boltzmann machine*

Naive Bayes classifier

- Naive Bayes classifier is a well-known simple classifier
- It was found to be effective on a number tasks, primarily in *document classification*
- Popularized by practical spam detection applications
- *Naive* part comes from a strong independence assumption
- *Bayes* part comes from the use of Bayes' formula for inverting conditional probabilities
- However, learning is (typically) 'not really' Bayesian

Naive Bayes: estimation

- Given a set of features $\mathbf{x}$, we want to know the class y of the object we want to classify
- At prediction time we pick the class, $\hat{y}$

$$\hat{y} = \arg\max_{y} P(y \mid \mathbf{x})$$

- Instead of directly estimating the conditional probability, we invert it using the Bayes' formula

$$\hat{y} = \arg\max_{y} \frac{P(\mathbf{x} \mid y)P(y)}{P(\mathbf{x})} = \arg\max_{y} P(\mathbf{x} \mid y)P(y)$$

- Now the task becomes estimating $P(\mathbf{x} \mid y)$ and $P(y)$

Naive Bayes: estimation (cont.)

- Class distribution, $P(y)$, is estimated using the MLE on the training set
- With many features, $\mathbf{x} = (x_1, x_2, \ldots x_n)$, $P(\mathbf{x} \mid y)$ is difficult to estimate
- Naive Bayes estimator makes a conditional independence assumption: given the class, we assume that the features are independent of each other

$$P(\mathbf{x} \mid y) = P(x_1, x_2, \ldots x_n \mid y) = \prod_{i=1}^{n} P(x_i \mid y)$$

Naive Bayes: estimation (cont.)

- The probability distributions $P(x_i \mid y)$ and $P(y)$ are typically estimated using MLE (count and divide)
- A *smoothing* technique may be used for unknown features (e.g., words)
- Note that $P(x_i \mid y)$ can be

binomial e.g, whether a word occurs in the document or not
categorical e.g, estimated using relative frequency of words
continuous the data is distributed according to a known distribution

Naive Bayes

a simple example: spam detection

Training data:

features present	label
good book	NS
now book free	S
medication lose weight	S
technology advanced book	NS
now advanced technology	S

- A test instance: {book, technology}
- Another one: {good, medication}

$P(S) = 3/5$, $P(NS) = 2/5$

w	$P(w \mid S)$	$P(w \mid NS)$
medication	1/3	0
free	1/3	0
technology	1/3	1/2
advanced	1/3	1/2
book	1/3	2/2
now	2/3	0
lose	1/3	0
weight	1/3	0
good	0	1/2

Classifying classification methods

another short digression

- Some classification algorithms are non-probabilistic, discriminative: they return a label for a given input. Examples: perceptron, SVMs, decision trees
- Some classification algorithms are discriminative, probabilistic: they estimate the conditional probability distribution $p(\mathbf{c} \mid \mathbf{x})$ directly. Examples: logistic regression, (most) neural networks
- Some classification algorithms are generative: they estimate the joint distribution $p(\mathbf{c}, \mathbf{x})$. Examples: naive Bayes, Hidden Markov Models, (some) neural models

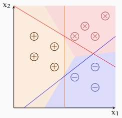
More than two classes

- Some algorithms can naturally be extended to handle multiple class labels
- Any binary classifier can be turned into a k-way classifier by

OvR one-vs-rest or one-vs-all
  - train k classifiers: each learns to discriminate one of the classes from the others
  - at prediction time the classifier with the highest confidence wins
  - needs a confidence score from the base classifiers

OvO one-vs-one
  - train $\frac{k(k-1)}{2}$ classifiers: each learns to discriminate a pair of classes
  - decision is made by (weighted) majority vote
  - works without need for confidence scores, but needs more classifiers

One vs. Rest

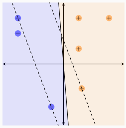


- For 3 classes, we fit 3 classifiers separating one class from the rest
- Some regions of the feature space will be ambiguous
- We can assign labels based on probability or weight value, if classifier returns one
- One-vs.-one and majority voting is another option

More classification methods ...

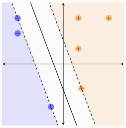
- Classification is a well-studied topic in ML, with a large range of applications
- There are many different approaches
- In most cases you can 'plug' a classification algorithm instead of another, treating classifiers as 'black boxes'
- You should, however, understand the methods you use: you may not be able to use them properly if you do not understand them
- One-slide introduction to some of the methods we did not cover starts on the next slide
- We will return to some specialized methods later in this course

Maximum-margin methods (e.g., SVMs)



- In perceptron, we stopped whenever we found a linear discriminator
- Maximum-margin classifiers seek a discriminator that maximizes the margin
- SVMs have other interesting properties, and they have been one of the best 'out-of-the-box' classifiers for many problems

Maximum-margin methods (e.g., SVMs)



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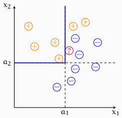
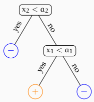
A quick survey of some solutions

Decision trees

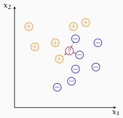




- Note that the decision boundary is non-linear

A quick survey of some solutions

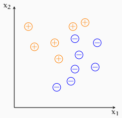
Instance/memory based methods

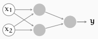


- No training: just memorize the instances
- During test time, decide based on the k nearest neighbors
- Like decision trees, kNN is non-linear
- It can also be used for regression

A quick survey of some solutions

Artificial neural networks





Measuring success in classification

Accuracy

- In classification, we do not care (much) about the average of the error function
- We are interested in how many of our predictions are correct
- Accuracy measures this directly

$$\text{accuracy} = \frac{\text{number of correct predictions}}{\text{total number of predictions}}$$

Accuracy may go wrong

- Think about a ‘dummy’ search engine that always returns an empty document set (no results found)
- If we have
  - 1 000 000 documents
  - 1000 relevant documents (related to the terms in the query)

  the accuracy is:

  $$\frac{999\,000}{1\,000\,000} = 99.9\,\%$$
- In general, if our class distribution is *skewed*, or *imbalanced*, accuracy will be a bad indicator of success

Measuring success in classification

Precision, recall, F-score

$$\text{precision} = \frac{TP}{TP + FP}$$

$$\text{recall} = \frac{TP}{TP + FN}$$

$$F_1\text{-score} = \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$$

		predicted	
		positive	negative
true value	pos.	TP	FN
	neg.	FP	TN

Example: back to the ‘dummy’ search engine

- For a query
  - 1 000 000 documents
  - 1000 relevant documents

$$\text{accuracy} = \frac{999\,000}{1\,000\,000} = 99.9\,\%$$

$$\text{precision} = \frac{0}{0} = 0\,\% \text{ (undefined, common convention)}$$

$$\text{recall} = \frac{0}{1\,000\,000} = 0\,\%$$

Precision and recall are asymmetric,
the choice of the ‘positive’ class is important.

Classifier evaluation: another example

Consider the following two classifiers:

		predicted		predicted	
		positive	negative	positive	negative
true value	pos.	7	3	1	9
	neg.	9	1	3	7

Accuracy both 8/20 = 0.4
Precision 7/16 = 0.44 and 1/4 = 0.25
Recall 7/10 = 0.7 and 1/10 = 0.1
F-score 0.54 and 0.14

Multi-class evaluation

- For multi-class problems, it is common to report average precision/recall/f-score
- For C classes, averaging can be done two ways:

$$\text{precision}_M = \frac{\sum_i^C \frac{TP_i}{TP_i + FP_i}}{C} \qquad \text{recall}_M = \frac{\sum_i^C \frac{TP_i}{TP_i + FN_i}}{C}$$

$$\text{precision}_\mu = \frac{\sum_i^C TP_i}{\sum_i^C TP_i + FP_i} \qquad \text{recall}_\mu = \frac{\sum_i^C TP_i}{\sum_i^C TP_i + FN_i}$$

(M = macro, μ = micro)

- The averaging can also be useful for binary classification, if there is no natural positive class

Confusion matrix

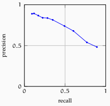
- A confusion matrix is often useful for multi-class classification tasks

		predicted		
		negative	neutral	positive
true value	negative	10	2	0
	neutral	3	12	7
	positive	4	8	7

- Are the classes balanced?
- What is the accuracy?
- What is per-class, and averaged precision/recall?

Precision–recall trade-off

- Increasing precision (e.g., by changing a hyperparameter) results in decreasing recall
- Precision–recall graphs are useful for picking the correct models
- *Area under the curve* (AUC) is another indication of success of a classifier



Performance metrics a summary

- Accuracy does not reflect the classifier performance when class distribution is skewed
- Precision and recall are binary and asymmetric
- For multi-class problems, calculating accuracy is straightforward, but others measures need averaging
- These are just the most common measures, there are more
- You should understand what these metrics measure, and use/report the metric that is useful for the purpose

Summary

- We discussed three basic classification techniques: perceptron, logistic regression, naive Bayes
- We left out many others: SVMs, decision trees, …
- We also did not discuss a few other interesting cases, including *multi-label* classification
- Reading suggestion: James et al. (2023, ch.4), Jurafsky and Martin (2009, ch.4&5, draft 3rd edition)

Next

- Unsupervised learning: clustering
- Reading suggestion: James et al. (2023, section 12.4)